

UNSTEADY STATE HEAT TRANSFER

PURPOSE

The purpose is to examine the temperature distribution along the brass rod maintained at two fixed temperatures and the heat loss from this rod.

THEORY

A summary of the relevant basic information and a computer program for the theoretical solution are given.

THE EXPERIMENTAL SETUP

It is a cylindrical brass rod (diameter: 4cm, length: 80cm) that is heated by electric power from its end sides and placed in a thermometer in various positions along its length.

EXPERIMENTAL METHOD

The two ends of the cylindrical brass rod are heated to 150 °C with electrical resistances and are kept constant at this temperature. Every 1 minutes, the temperatures are read from the thermometers placed along the rod at the same time. This process lasts up to 10 minutes after the thermometers in the rod have reached 150 °C. The obtained data are recorded in the table. The ambient temperature is also measured.

CALCULATIONS

1. Calculate the average heat transfer coefficient (h) between the cylinder surface and the air under the following conditions.
 - a) Steady state,
 - b) The air is completely stagnant (no convection currents),
 - c) The cylinder surface temperature (T_w) is constant,
 - d) The temperature of the air is equal to the temperature (T_∞) at $r = 785R$ from the outer surface of the cylinder.
2. Calculate the average heat transfer coefficient between the cylinder surface and the air for a natural convection state using an average temperature for the rod. Calculate the heat loss from the rod by natural convection.
3. If it is assumed that $dT / dx = 0$ should be in the middle of the rod due to symmetry, half of the rod can be considered as a perfectly isolated cylindrical fin held at one end T_s temperature and the other end perfectly insulated. Under these circumstances, for steady state.
 - a) Determine the temperature distribution along the rod (8cm interval).
 - b) Calculate the heat loss from the rod to the environment.
4. Using the first two terms of the series sum in the expression of the temperature distribution obtained for unsteady state heat transfer from horizontal rod whose ends are kept at constant temperature, obtained the temperature distribution along the bar (8cm interval) at $t=10$. minute. Compare these results with your experimental data on the same graph. Also, add the steady state data you obtained in (3) on the same graph.

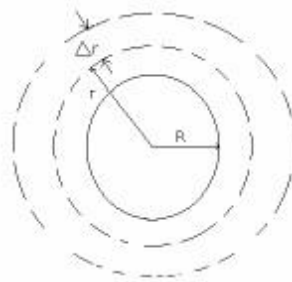
NOTE: 1) In the appendix, a sample program written in BASIC that allows to obtain the temperature distribution by using more terms of the series sum of the expression derived for unsteady state heat transfer from horizontal rod whose ends are kept at constant temperature and the temperature distribution graph obtained for different times are given. You can use a similar program for the solution.

2) Temperature distribution for the unsteady state (parabolic partial differential equation, $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$) can also be obtained by using numerical methods (explicit method, Crank-Nicholson Methods etc.) in MATLAB.

HEAT TRANSFER THROUGH STAGNANT FLUID ENVIRONMENT FROM CYLINDER SURFACE

Assumptions:

- Cylinder outer surface temperature $T_w = \text{constant}$
- Steady state
- In the case of $\partial r = 785R$ $T = T_\infty$



Energy equation for shell:

$$\left\{ (2\pi L) \left(-k \frac{dT}{dr} \right) \right\} \Big|_r - \left\{ (2\pi L) \left(-k \frac{dT}{dr} \right) \right\} \Big|_{r+\Delta r} = 0$$

If dividing this equation by $(2\pi L \Delta r)$ and taking the limit in $\Delta r \rightarrow 0$,

$$\lim_{\Delta r \rightarrow 0} \frac{\left(r \frac{dT}{dr} \right) \Big|_{r+\Delta r} - \left(r \frac{dT}{dr} \right) \Big|_r}{\Delta r} = 0 \Rightarrow \frac{d \left\{ r \frac{dT}{dr} \right\}}{dr} = 0$$

$$r \frac{dT}{dr} = C_1 \Rightarrow dT = C_1 \frac{dr}{r} \Rightarrow T = C_1 \ln r + C_2$$

Boundary Condition 1: $r=R$; $T=T_w \Rightarrow T_w = C_1 \ln R + C_2$

Boundary Condition 2: $r=785R$; $T=T_\infty \Rightarrow T_\infty = C_1 \ln(785R) + C_2$

In this case;

$$(T_w - T_\infty) = C_1 (\ln R - \ln 785R) \Rightarrow C_1 = \frac{T_w - T_\infty}{\ln \frac{R}{785R}} \Rightarrow C_1 = \frac{T_w - T_\infty}{-6,666}$$

$q|_{r=R} = ?$

$$q|_{r=R} = (2\pi RL) \left(-k \frac{dT}{dr} \right) \Big|_{r=R} = -2\pi RLk \frac{C_1}{r} \Big|_{r=R} = -2\pi RLk \frac{(T_w - T_\infty)}{(-6,666)R} = \frac{\pi Lk}{3,333} (T_w - T_\infty)$$

Using Newton's cooling law,

$$q|_{r=R} = \frac{\pi L k}{3,333} (T_w - T_\infty) \equiv h(T_w - T_\infty)(2\pi RL) \Rightarrow \frac{k}{3,333} = hD \Rightarrow \frac{hD}{k} = 0,3 \Rightarrow Nu = 0,3$$

NOTE: For convective heat transfer of vertical surface from single cylinder;

$$Nu = 0,3 + \frac{0,62 Re^{\frac{1}{2}} Pr^{\frac{1}{3}}}{\left[1 + (0,4/Pr)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \left[1 + \left(\frac{Re}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

When the velocity of the fluid approaches zero ($Re \rightarrow 0$); ($Nu \rightarrow 0,3$), this result is the same as that obtained above.

HEAT TRANSFER BY NATURAL CONVECTION FROM HORIZONTAL CYLINDER

$$Nu = cRa^n$$

Nusselt Number $Nu = \frac{hD}{k}$

Prandtl Number $Pr = \frac{c_p \mu}{k}$

Grashof Number $Gr = \frac{g\beta(T_w - T_\infty)D^3}{\nu^2}$

Rayleigh Number $Ra = Gr \cdot Pr$

Ra	c	n
10^{-10} - 10^{-2}	0,675	0,058
10^{-2} - 10^2	1,02	0.148
10^2 - 10^4	0,85	0,188
10^4 - 10^7	0,48	0,25
10^7 - 10^{12}	0,125	0,333

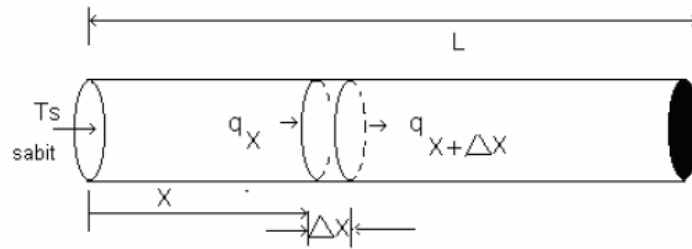
The properties of the fluids are taken as $T_f = (T_w + T_\infty) / 2$ at the film temperature.

T_w : Cylinder outer surface temperature

T_∞ : Ambient temperature

HEAT LOSS FROM CYLINDRICAL FIN (STEADY STATE)

One end is very well insulated, the other end is constant at (T_s) temperature. At the isolated end, $(dT/dx)_{x=L}=0$.



$$\text{Perimeter} = P = \pi D; \text{Area} = A = \pi D^2/4$$

$$q|_x - q|_{x+\Delta x} - h(T - T_\infty)P\Delta x = 0$$

$$\left[-kA \frac{dT}{dx} \right]_x - \left[-kA \frac{dT}{dx} \right]_{x+\Delta x} - h(T - T_\infty)P\Delta x = 0$$

$$\left[-kA \frac{dT}{dx} \right]_x - \left[-kA \frac{dT}{dx} \right]_x + \frac{d^2T}{dx^2} \Delta x - h(T - T_\infty)P\Delta x = 0$$

$$\frac{d^2T}{dx^2} = \frac{hP}{kA} (T - T_\infty)$$

$$\theta = (T - T_\infty); \quad m^2 = \frac{hP}{kA} \quad \Rightarrow \quad \frac{\partial^2 \theta}{\partial x^2} = m^2 \theta$$

The general solution of this differential equation is: $\theta = C_1 e^{-mx} + C_2 e^{+mx}$

Boundary Condition 1: $x=0$; $T = T_s$; $\theta = \theta_s = T_s - T_\infty$

Boundary Condition 2: $x=L$; $\frac{dT}{dx}|_{x=L} = 0$; $\frac{d\theta}{dx}|_L = 0$

From the general solution; $\frac{d\theta}{dx} = -C_1 m e^{-mx} + C_2 m e^{+mx}$

$$\text{B.C.2} \Rightarrow 0 = -C_1 m e^{-mx} + C_2 m e^{+mx}$$

$$\text{B.C.1} \Rightarrow \theta = C_1 + C_2$$

The common solution of these two equations is:

$$C_1 = \theta_s \frac{e^{mL}}{(e^{mL} + e^{-mL})} ; \quad C_2 = \theta_s \frac{e^{-mL}}{(e^{mL} + e^{-mL})}$$

$$\text{Solution: } \frac{\theta}{\theta_s} = \frac{e^{m(L-x)} + e^{-m(L-x)}}{e^{mL} + e^{-mL}}$$

$$\cosh(y) = \frac{e^y + e^{-y}}{2}$$

$$\frac{\theta}{\theta_s} = \frac{\cosh m(L-x)}{\cosh mL} \rightarrow \text{Temperature Distribution}$$

Heat loss to the environment:

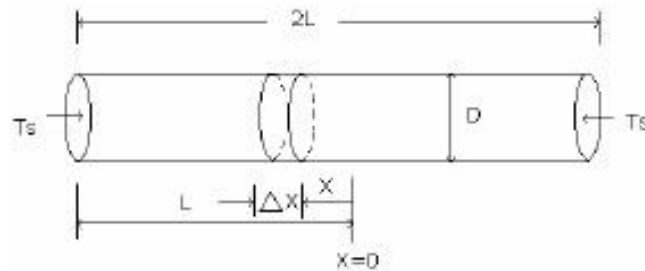
$$q = -kA \left. \frac{dT}{dx} \right|_{x=0} = \int_0^L h(T - T_\infty) \pi D dx$$

From the temperature distribution expression; $\frac{dT}{dx} = \theta_s \frac{\sinh m(L-x)}{\cosh mL} (-m)$

$$\left. \frac{dT}{dx} \right|_{x=0} = -m\theta_s \tanh mL \Rightarrow q = kAm(T_s - T_\infty) \tanh mL$$

NOTE: $mL = \sqrt{Bi}$; $Bi = \frac{hl}{k}$, Biot Number; $l = \frac{PL^2}{A}$, Characteristic dimension of the system

UNSTEADY STATE HEAT TRANSFER FROM HORIZONTAL ROD WHOSE ENDS ARE KEPT AT CONSTANT TEMPERATURE



$$\text{Perimeter} = P = \pi D; \text{Area} = A = \pi D^2/4$$

If the energy balance is written for the shell, paying attention to the fact that the heat transfer by the conduction is in the -x direction;

$$-\xi|_{x+\Delta x} - [-\xi|_x] - h(T - T_\infty)P\Delta x = A\Delta x \rho C_p \frac{dT}{dt}$$

$$-\left(-kA \left. \frac{dT}{dx} \right|_{x+\Delta x}\right) - \left[-\left(-kA \left. \frac{dT}{dx} \right|_x\right)\right] - h(T - T_\infty)P\Delta x = A\Delta x \rho C_p \frac{dT}{dt}$$

Dividing it by Δx and taking the limit when $\Delta x \rightarrow 0$;

$$\frac{\partial^2 T}{\partial x^2} - \frac{hP}{kA}(T - T_\infty) = \frac{\rho C_p}{k} \frac{\partial T}{\partial t}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial x^2} - \frac{hP}{A \rho C_p} (T - T_\infty)$$

If the dimensionless groups are expressed as follows and the above equation is rearranged;

$$\alpha = \frac{k}{\rho C_p}, \quad \beta = \frac{hP}{A \rho C_p}, \quad \theta = (T - T_\infty)$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial T}{\partial t}, \quad \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2} - \beta \theta$$

If we define a new variable and it will be defined as $\theta = (u)(e^{-nt})$ and if the above equation is rearranged;

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

Initial Condition : $t = 0 ; \quad u = (T_i - T_\infty) = \theta_i$

Boundary Condition 1: $x = +L ; \quad u = (T_s - T_\infty)e^{\beta t} = \theta_s e^{\beta t}$

Boundary Condition 2: $x = -L ; \quad u = (T_s - T_\infty)e^{\beta t} = \theta_s e^{\beta t}$

By taking advantage of these initial and boundary conditions, the temperature distribution can be obtained from the solution of the above partial differential equation.

$$\theta = \theta_s \frac{\cosh x(\beta/\alpha)^{1/2}}{\cosh L(\beta/\alpha)^{1/2}} - \frac{4\theta_s}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n e^{-\beta t - \left[\frac{\alpha(2n+1)^2 \pi^2 t}{4L^2}\right]} \cos \left[\frac{(2n+1)\pi x}{2L}\right]}{(2n+1) \left[1 + \frac{4\beta L^2}{(2n+1)^2 \pi^2 \alpha}\right]}$$

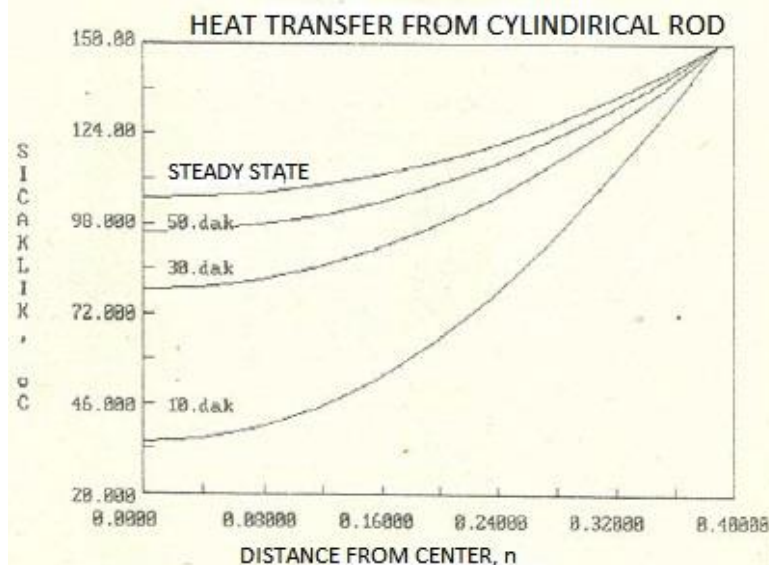
The first term on the right side of this equation is the solution for the “steady state”. By considering the difference in the -x direction and $(\beta/\alpha)^{0.5} = m$, it is seen that this term is the same as the temperature distribution obtained from steady state heat loss from cylindrical fin whose one end is isolated in the previous section.

Appendix: A sample program and output

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_LIST
10 REM BU PROGRAM İKİ UÇU T0 SICAKLIĞINDA TUTULAN BİR CUBUK BOYUNCA SICAKLIK D
AGILIMINI HESAPLAMAKTADIR. SI BİRİM SİSTEMİ KULLANILMIŞTIR.
20 REM TINF=ORTAM SICAKLIĞI
30 REM TI=CUBUK İLK SICAKLIĞI (=TINF)
40 REM L=CUBUK BOYUNUN YARISI
50 REM X=CUBUK MERKEZİNDEN OLAN UZAKLIK, m
60 REM TSS=KARARLI HAL SICAKLIK DEĞERİ, °C
70 REM TUS=KARARSIZ HAL SICAKLIK DEĞERİ, °C
80 REM D,K,RO,CP:CUBUGUN ÇAPİ, İSİ İLETKENLİĞİ, YÜZÖLÇÜSÜ, İSİ KAPASİTESİ
90 REM H=CUBUK YÜZEYİ İLE ORTAM ARASINDAKİ İSİ TRANSFER KATSAYISI
100 PI=22!/7!
110 READ D,K,RO,CP,H
120 DATA 0.042,120.,8522.,380.,8.
130 READ T0,TINF,L
140 DATA 150.,20.,0.39
150 NINT=10
160 FOR DAKİKA=10 TO 60 STEP 10
170 TIME=60*DAKİKA
180 P=PI*D
190 A=PI*D^2!/4!
200 ALPHA=K/(RO*CP)
210 BETA=H*P/(RO*CP*A)
220 M=SQR(BETA/ALPHA)
230 DEF FNSINH(X)=(EXP(X)-EXP(-X))/2!
240 DEF FNCOSH(X)=(EXP(X)+EXP(-X))/2!
250 XINT=L/NINT
260 X=0
270 TETA0=T0-TINF
280 PRINT "ZAMAN=" DAKİKA "DAKİKA"
290 PRINT
300 PRINT "X","S-S TEMP","U-S TEMP"
310 PRINT
320 FOR J=0 TO (NINT-1)
330 MX=M*X
340 ML=M*L
350 TETASS=TETA0+FNCOSH(MX)/FNCOSH(ML)
360 SUM=0
370 FOR N=0 TO 10
380 NUM=(-1!)^N*EXP((-BETA*TIME)-(ALPHA*((2!*N)+1!)^2!*PI^2!*TIME/(4!*L^2!)))*CO
S((2!*N)+1!)*PI*X/(2!*L)
390 DENUM=((2!*N)+1!)*(1!+(4!*BETA*L^2!/((2!*N)+1!)^2!*PI^2!*ALPHA))
400 SUM=SUM+NUM/DENUM
410 NEXT N
420 TETAUS=TETASS-(4!*TETA0/PI)*SUM
430 TSS=TETASS+TINF
440 TUS=TETAUS+TINF
450 PRINT X,TSS,TUS
460 X=X+XINT
470 NEXT J
480 PRINT L,T0
490 PRINT
500 NEXT DAKİKA
510 END
C

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[illegible]